# Sensitivity Analysis of the Empirical Model

The following is a sensitivity analysis for the empirical model for propeller noise taken from Chapter 12 of the textbook “Elements of Aviation Acoustics” by G.J.J Ruijgrok. The propeller noise equation is given as:

|  |  |
| --- | --- |
|  |  |
|  |  |
|  | |

# Input Identification of Regression Analysis Based Sound Models

Following table identifies the different inputs to the several flyover sound models delineated in the technical study “The Development of a Flyover Noise Prediction Technique Using Multiple Linear Regression Analysis” by Cessna Aircraft Co.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Model Number:** | | **1** | **2** | **3** | **4** | **5** | **7** | **8** | **9** | **10** |
| **Parameter** | **Unit** |  |  |  |  |  |  |  |  |  |
| Helical Mach | - | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Shaft HP | Hp |  |  |  |  |  | ✓ | ✓ | ✓ | ✓ |
| Helical Mach Squared | - |  |  |  |  |  | ✓ |  |  |  |
| Two or Three Blades ? | - |  |  | ✓ |  |  | ✓ | ✓ |  | ✓ |
| Turbocharger Enabled? | - |  |  |  |  |  | ✓ | ✓ |  | ✓ |
| Single or Twin engine? | - |  |  |  |  |  | ✓ |  |  | ✓ |

## Comparison between Models 7 and 10











# Codes for both Models

## Empirical Model

clc;clear;close all

%For this code, let us use the data from Chapter 14 examples of Gudmundson

%The model has been taken from Ref 18 - "Elements of aviation Acoustics"

%Starting with the piston engine noise:

n = 2600;%engine rotational speed in RPM

N = 4;%number of cylinders

f\_c = n/120;%cylinder firing frequency

f\_e = (N\*n)/120;%exhaust firing frequency

% According to Reference 61, the overall A-weighted level of the exhaust noise

% of an unmuffled piston engine at 150 m sideline can be estimated by:

P\_br = 310\*745.7;%engine shaft power, watts

L\_A = 8 + (14\*log10(P\_br)) %dBA

%Using data in Gudmundson - An airplane is powered by a two-bladed propeller

%whose diameter is 76 inches is driven by a 310 BHP engine

%Then moving on to propeller noise:

% For the prediction of far-field propeller noise, the following expres sion for the

% maximum sound pressure level can be used:

B = 3;%number of blades per propeller

n\_p = 2600;%propeller rotational speed (rpm)

D = 1.9304;%propeller diameter (m)

r = 50;%distance from propeller (m)

c = 340;%speed of sound(m/s)

M\_t = (pi\*D\*n\_p)/(60\*c);%tip mach number

SPL\_max = 83.4 + (15.3\*log10(P\_br)) - (20\*log10(D)) + (38.5\*M\_t) + ...

(-3\*(B - 2)) + (10\*log10(N)) - (20\*log10(r))

%Sensitivity Analysis for the propeller model

%First we will vary each input parameter indvidually to check its impact on

%the sound level while keeping the other inputs constant.

%Sensitivity to Number of Blades

figure(1)

B\_set = [2, 3, 4, 5, 6];

SPL\_max\_B = 83.4 + (15.3.\*log10(P\_br)) - (20.\*log10(D)) + (38.5.\*M\_t) + ...

(-3.\*(B\_set - 2)) + (10.\*log10(N)) - (20.\*log10(r))

plot(B\_set,SPL\_max\_B,'r','LineWidth',2);grid on

xlabel('Number of blades');ylabel('SPL\_{max} (dBA)');title('SPL variation with No. of Blades')

%Sensitivity to Engine Horse Power

figure(2)

P\_br\_set = [200:10:400].\*745.7;%watts

SPL\_max\_P\_br = 83.4 + (15.3.\*log10(P\_br\_set)) - (20.\*log10(D)) + (38.5.\*M\_t) + ...

(-3.\*(B - 2)) + (10.\*log10(N)) - (20.\*log10(r))

plot(P\_br\_set,SPL\_max\_P\_br,'g','LineWidth',2);grid on

xlabel('engine shaft power (watts)');ylabel('SPL\_{max} (dBA)');title('SPL variation with engine shaft power')

%Sensitivity to Propeller RPM i.e. tip mach

figure(3)

n\_p\_set = 1000:100:3000

M\_t\_set = (pi.\*D.\*n\_p\_set)./(60.\*c);%tip mach number

SPL\_max\_n\_p = 83.4 + (15.3.\*log10(P\_br)) - (20.\*log10(D)) + (38.5.\*M\_t\_set) + ...

(-3.\*(B - 2)) + (10.\*log10(N)) - (20.\*log10(r))

plot(n\_p\_set,SPL\_max\_n\_p,'b','LineWidth',2);grid on

xlabel('Propeller RPM');ylabel('SPL\_{max} (dBA)');title('SPL variation with propeller RPM')

%Sensitivity to Propeller Diameter i.e. also affects tip mach

figure(4)

D\_set = 0.5:0.1:2;%propeller diameter (m)

M\_t\_set\_2 = (pi.\*D\_set.\*n\_p)./(60.\*c);%tip mach number

SPL\_max\_D = 83.4 + (15.3.\*log10(P\_br)) - (20.\*log10(D\_set)) + (38.5.\*M\_t\_set\_2) + ...

(-3.\*(B - 2)) + (10.\*log10(N)) - (20.\*log10(r))

plot(D\_set,SPL\_max\_D,'c','LineWidth',2);grid on

xlabel('Propeller Diameter (m)');ylabel('SPL\_{max} (dBA)');title('SPL variation with propeller diameter')

%Sensitivity to distance From Propeller

figure(5)

r\_set = 0:10:100;%distance from propeller (m)

SPL\_max\_r = 83.4 + (15.3.\*log10(P\_br)) - (20.\*log10(D)) + (38.5.\*M\_t) + ...

(-3.\*(B - 2)) + (10.\*log10(N)) - (20.\*log10(r\_set))

plot(r\_set,SPL\_max\_r,'k','LineWidth',2);grid on

xlabel('distance from propeller (m)');ylabel('SPL\_{max} (dBA)');title('SPL variation with distance from propeller')

## Regression Models

%Following are the regression based models from the paper:

% "The Development of a Flyover Noise Prediction Technique Using Multiple

% Linear Regression Analysis"

clc;clear variables;close all

%Defining the State Conditions at different altitudes

%specifying all of the altitudes

alt\_ft = 1000:1000:35000;

alt\_m = alt\_ft \* 0.3048;

%Temp in K, Sound Speed in m/s, Pressure in Pa, rho(h) in kg/m3

[T\_si,a\_si,P\_si,rho\_si] = atmoscoesa(alt\_m);

%converting atmospheric values to english uni`ts

T\_eng = 1.8\*T\_si;%rankine

a = 3.28084\*a\_si;%ft/s

P = 0.02088547\*P\_si;%lb/ft^2

rho = 0.00194032\*rho\_si;%slugs/ft^3

%MODEL # 1 - (Single Engine Aircraft)

X\_1 = 0.79;%Helical Mach

Y\_1 = 12.7506 + (75.6219\*X\_1);%dBA

%MODEL # 2 - (Twin Engine Aircraft)

X\_2 = 0.78;%Helical Mach

Y\_2 = 25.99 + (65.0586\*X\_2);%dBA

%MODEL # 3 - (Single and Twin Engine)

X\_3\_1 = 0.78;%Helical Mach

X\_3\_2 = 0;% 0 for single and 1 for double prop

Y\_3 = 13.2314 + (75.0445\*X\_3\_1) + (4.3295\*X\_3\_2);%dBA

%MODEL # 4 - (Single Engine Aircraft)

X\_4 = 0.78;%Helical Mach

Y\_4 = 86.7697 + (137.8972\*X\_4);%dBA

%MODEL # 5 - (Single Engine Aircraft)

X\_5 = 0.78;%Helical Mach

Y\_5 = 60.8837 + (0.00481\*X\_5);%dBA

%MODEL # 6 - ILLEGIBLE

%MODEL # 8 - (Single Engine Aircraft)

X\_8\_1 = 250;%BHP

X\_8\_2 = 0.78;%Helical Mach

X\_8\_3 = 0;%0 for 2 blade, 1 for 3 blade

X\_8\_4 = 0;%0 for non-turbo, 1 for turbo

Y\_8 = 30.5646 + (0.00942\*X\_8\_1) + (49.9636\*X\_8\_2) + (2.4494\*X\_8\_3) + (0.4552\*X\_8\_4);

%MODEL # 9 - (Twin Engine Aircraft)

X\_9\_1 = 250;%BHP

X\_9\_2 = 0.78;%Helical Mach

Y\_9 = 5.2566 + (0.01428\*X\_9\_1) + (84.2969\*X\_9\_2);

%MODEL # 10 -

X\_10\_1 = 250;%BHP

X\_10\_2 = 0.78;%Helical Mach

X\_10\_3 = 0;%0 for 2 blade, 1 for 3 blade

X\_10\_4 = 0;%0 for non-turbo, 1 for turbo

X\_10\_5 = 0;%0 for single engine, 1 for twin engine

Y\_10 = 28.8194 + (0.00678\*X\_10\_1) + (52.6543\*X\_10\_2) + (2.8333\*X\_10\_3)...

+ (0.2603\*X\_10\_4) + (2.5742\*X\_10\_5);

%MODEL # 7 - (Sing and Twin Engine Aircraft)

X\_7\_1 = 250;%BHP

X\_7\_2 = 0.78;%Helical Mach

X\_7\_3 = X\_7\_2^2;%Helical Mach Squared

X\_7\_4 = 0;%0 for 2 blade, 1 for 3 blade

X\_7\_5 = 0;%0 for non-turbo, 1 for turbo

X\_7\_6 = 0;%0 for single engine, 1 for twin engine

Y\_7 = 31.3920 + (0.0067\*X\_7\_1) + (46.1576\*X\_7\_2) + (4.2376\*X\_7\_3) + ...

(2.5981\*X\_7\_4) + (0.2577\*X\_7\_5) + (2.6106\*X\_7\_6);

%COMPARISONS BETWEEN MODELS 7 & 10

%Helical Tip Mmach Variance Comparison:

%Single engine (250 hp), 2 blade, non turbo

figure(1)

M\_hel\_Set = 0.2:0.05:0.8;

Y\_7\_Mach\_1 = 31.3920 + (0.0067.\*250) + (46.1576.\*M\_hel\_Set) + (4.2376.\*(M\_hel\_Set.^2)) + ...

(2.5981.\*0) + (0.2577.\*0) + (2.6106.\*0);

Y\_10\_Mach\_1 = 28.8194 + (0.00678\*250) + (52.6543\*M\_hel\_Set) + (2.8333\*0)...

+ (0.2603\*0) + (2.5742\*0);

%Twin Engine, 3 blade, turbo

Y\_7\_Mach\_2 = 31.3920 + (0.0067.\*250) + (46.1576.\*M\_hel\_Set) + (4.2376.\*(M\_hel\_Set.^2)) + ...

(2.5981.\*1) + (0.2577.\*1) + (2.6106.\*1);

Y\_10\_Mach\_2 = 28.8194 + (0.00678\*250) + (52.6543\*M\_hel\_Set) + (2.8333\*1)...

+ (0.2603\*1) + (2.5742\*1);

plot(M\_hel\_Set,Y\_7\_Mach\_1,'r','LineWidth',2);grid on;hold on

plot(M\_hel\_Set,Y\_10\_Mach\_1,'g','LineWidth',2)

plot(M\_hel\_Set,Y\_7\_Mach\_2,'b','LineWidth',2);

plot(M\_hel\_Set,Y\_10\_Mach\_2,'k','LineWidth',2)

xlabel('Helical Tip Mach');ylabel('SPL\_{max} (dBA)');title('comparison for varying mach numbers')

legend('Model 7 (Single engine, 2 blade, non turbo)', 'Model 10 (Single engine, 2 blade, non turbo)',...

'Model 7 (Twin Engine, 3 blade, turbo)', 'Model 10 (Twin Engine, 3 blade, turbo)','Location','NorthWest')

xlim([0.2 0.8])

%Shaft Horse Power Variance Comparison:

%Mach 0.78, 2 blade, non turbo

figure(2)

M\_hel\_Set = 0.78;

Power\_Set = 100:10:500;%hp

Y\_7\_Power\_1 = 31.3920 + (0.0067.\*Power\_Set) + (46.1576.\*M\_hel\_Set) + (4.2376.\*(M\_hel\_Set.^2)) + ...

(2.5981.\*0) + (0.2577.\*0) + (2.6106.\*0);

Y\_10\_Power\_1 = 28.8194 + (0.00678.\*Power\_Set) + (52.6543.\*M\_hel\_Set) + (2.8333.\*0)...

+ (0.2603.\*0) + (2.5742.\*0);

%Twin Engine, 3 blade, turbo

Y\_7\_Power\_2 = 31.3920 + (0.0067.\*Power\_Set) + (46.1576.\*M\_hel\_Set) + (4.2376.\*(M\_hel\_Set.^2)) + ...

(2.5981.\*1) + (0.2577.\*1) + (2.6106.\*1);

Y\_10\_Power\_2 = 28.8194 + (0.00678\*Power\_Set) + (52.6543\*M\_hel\_Set) + (2.8333\*1)...

+ (0.2603\*1) + (2.5742\*1);

plot(Power\_Set,Y\_7\_Power\_1,'r','LineWidth',2);grid on;hold on

plot(Power\_Set,Y\_10\_Power\_1,'g','LineWidth',2)

plot(Power\_Set,Y\_7\_Power\_2,'b','LineWidth',2);

plot(Power\_Set,Y\_10\_Power\_2,'k','LineWidth',2)

xlabel('Shaft Power (hp)');ylabel('SPL\_{max} (dBA)');title('comparison for varying shaft powers')

legend('Model 7 (Single engine, 2 blade, non turbo)', 'Model 10 (Single engine, 2 blade, non turbo)',...

'Model 7 (Twin Engine, 3 blade, turbo)', 'Model 10 (Twin Engine, 3 blade, turbo)','Location','NorthWest')

%Setting up comparisons via models 7 and 10 w.r.t Diameter, RPM, and Speed

%Assuming configuration through all comparisons to be -> 250 hp, 2 blade, non turbo

%forward velocity of 50 ft/s if unvaried, Diameter of 6.33333 ft if unvaried

%Forward Speed:

figure(3)

alt = 1;

R = 6.3333/2; %ft

RPM = 2600;%rpm

RPM\_rads = RPM\*0.1047198;%rad/s

V\_forward = 50:10:250;%ft/s

V\_hel\_2 = sqrt((V\_forward.^2) + ((RPM\_rads.\*R).^2));

M\_hel\_Set\_2 = V\_hel\_2./a(alt)

Y\_7\_Forward = 31.3920 + (0.0067.\*250) + (46.1576.\*M\_hel\_Set\_2) + (4.2376.\*(M\_hel\_Set\_2.^2)) + ...

(2.5981.\*0) + (0.2577.\*0) + (2.6106.\*0);

Y\_10\_Forward = 28.8194 + (0.00678\*250) + (52.6543\*M\_hel\_Set\_2) + (2.8333\*0)...

+ (0.2603\*0) + (2.5742\*0);

plot(V\_forward,Y\_7\_Forward,'c','LineWidth',2);grid on;hold on

plot(V\_forward,Y\_10\_Forward,'m','LineWidth',2)

legend('Model 7 (Single engine, 2 blade, non turbo)',...

'Model 10 (Single engine, 2 blade, non turbo)')

xlabel('Shaft Power (hp)');ylabel('SPL\_{max} (dBA)');title('comparison for varying Forward Speeds')

%Prop Diameter

figure(4)

R\_set = (2:1:7)./2; %ft

RPM = 2600;%rpm

RPM\_rads = RPM\*0.1047198;%rad/s

V\_forward = 50;%ft/s

V\_hel\_3 = sqrt((V\_forward.^2) + ((RPM\_rads.\*R\_set).^2));

M\_hel\_Set\_3 = V\_hel\_3./a(alt)

Y\_7\_Rad = 31.3920 + (0.0067.\*250) + (46.1576.\*M\_hel\_Set\_3) + (4.2376.\*(M\_hel\_Set\_3.^2)) + ...

(2.5981.\*0) + (0.2577.\*0) + (2.6106.\*0);

Y\_10\_Rad = 28.8194 + (0.00678\*250) + (52.6543\*M\_hel\_Set\_3) + (2.8333\*0)...

+ (0.2603\*0) + (2.5742\*0);

plot(R\_set,Y\_7\_Rad,'r','LineWidth',2);grid on;hold on

plot(R\_set,Y\_10\_Rad,'g','LineWidth',2)

legend('Model 7 (Single engine, 2 blade, non turbo)',...

'Model 10 (Single engine, 2 blade, non turbo)')

xlabel('Prop Radii');ylabel('SPL\_{max} (dBA)');title('comparison for varying Prop Radii')

%Propeller RPM

figure(5)

R\_set = 6.33333/2; %ft

RPM\_set = 1000:100:2700;%rpm

RPM\_rads = RPM\_set.\*0.1047198;%rad/s

V\_forward = 50;%ft/s

V\_hel\_4 = sqrt((V\_forward.^2) + ((RPM\_rads.\*R\_set).^2));

M\_hel\_Set\_4 = V\_hel\_4./a(alt)

Y\_7\_RPM = 31.3920 + (0.0067.\*250) + (46.1576.\*M\_hel\_Set\_4) + (4.2376.\*(M\_hel\_Set\_4.^2)) + ...

(2.5981.\*0) + (0.2577.\*0) + (2.6106.\*0);

Y\_10\_RPM = 28.8194 + (0.00678\*250) + (52.6543\*M\_hel\_Set\_4) + (2.8333\*0)...

+ (0.2603\*0) + (2.5742\*0);

plot(RPM\_set,Y\_7\_RPM,'b','LineWidth',2);grid on;hold on

plot(RPM\_set,Y\_10\_RPM,'k','LineWidth',2)

legend('Model 7 (Single engine, 2 blade, non turbo)',...

'Model 10 (Single engine, 2 blade, non turbo)')

xlabel('RPM');ylabel('SPL\_{max} (dBA)');title('comparison for varying RPMs')